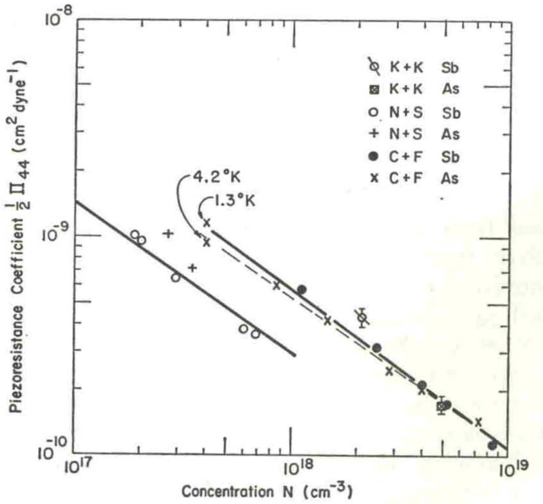

FIG. 6. Low stress piezoresistance coefficient $\frac{1}{2}\Pi_{44}$ as a function of donor concentration. The dashed line indicates the theoretically expected slope $\Pi_{44}\propto N^{-2/3}$. The letters behind the data symbols stand for the investigators: $K+K$=Koenig and Katz (Ref. 5 of text), $N+S$=Nakamura and Sasaki (Ref. 16 of text), $C+F$ =present authors (see also Ref. 6 of text).

The Hall coefficients obtained at 300 or 77°K were used to determine N and the mobilities in this figure because the Hall coefficients could not be measured at large stresses with the present apparatus. Since the Hall coefficient becomes temperature-dependent for $N<8\times10^{17}$ cm^{-3}, the plotted curves do not represent the true Hall mobilities and the abscissa is not the Hall concentration in this lower concentration range. The curves shown in Fig. 7, therefore, fall off much more rapidly with decreasing N than those of Furukawa.[12] The vertical arrows indicate the critical concentrations N_c at which the thermal activation energy ϵ_2 of impurity conduction vanishes.[3,6] The different values of N_c and the different behavior of the mobility curves below $N=8\times10^{17}$ cm^{-3} for the 4 and 1 valley cases and for Sb and As doping may be explained qualitatively as being due to the different effects of the valley-orbit splitting energies on the donor wave functions.

At the higher concentrations $N>10^{18}$ cm^{-3} we observe the following:

1. The concentration dependence of the mobility components is different for Sb and As doping. At zero stress and 1.2°K, for example, one finds for Sb, $\mu(4)\propto N^{-0.19}$; and for As, $\mu(4)\propto \mathrm{N}^{-0.22}$. This concentration dependence is significantly less than the $N^{-0.5}$ dependence obtained at 4.2°K from the free carrier absorption at a wavelength of 2.4 μ.[17,18]

2. The mobilities begin to decrease with increasing N and show a simple power law dependence on N at a lower concentration for As than for Sb. This is surprising since one would expect the onset of "metallic" conduction to occur at higher concentrations for As than for Sb for the same reason[3] which causes the difference in N_c.

3. The mobilities $\mu(4)$ of unstressed Ge doped with As are lower by a factor of about 1.4 than those of Ge doped with Sb. The ratio of $\mu_{\parallel}(1)$ of Sb to $\mu_{\parallel}(1)$ of As (orientation F) is as large as 1.9 at large concentrations. For the transverse direction (orientation G) the ratio $\mu_{\perp}(1)$ of Sb to $\mu_{\perp}(1)$ of As is about 1.3.

4. The low stress piezoresistance coefficient Π_{44} is the same for Sb and As doping within the experimental accuracy (see Fig. 6). For $N>10^{18}$ cm^{-3}, the concentration dependence is close to $\Pi_{44}\propto N^{-2/3}$ as expected from theory.

5. In contrast to the case of Sb doping the piezoresistance of As-doped Ge decreases beyond the saturation stress X_s and approaches a constant value only at higher stresses. This decrease is larger for the high mobility orientations G and D than for the low mobility orientations F and C.

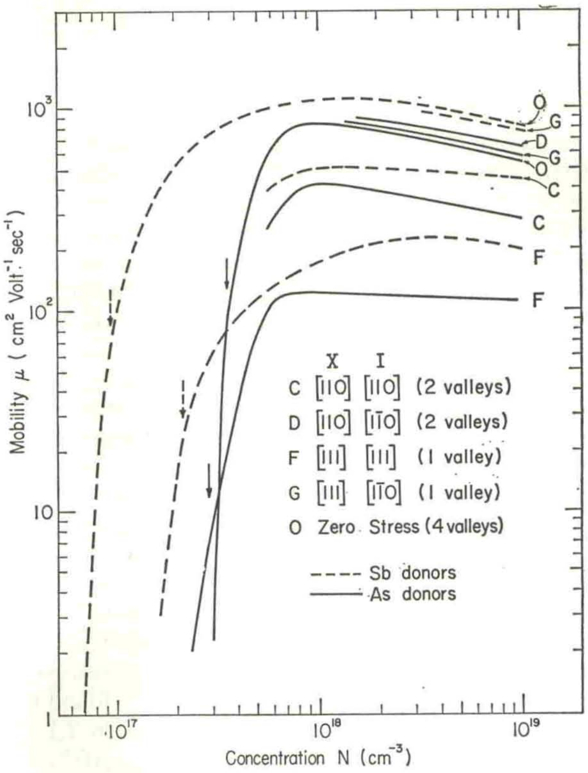

FIG. 7. Mobility components along the current direction in arrangements C, D, F, and G at 1.2°K are plotted versus concentration. These components were determined from the saturation values of the piezoresistance in these arrangements and the Hall coefficient as measured in the exhaustion region. The arrows indicate the critical concentrations N_c at which the activation energy for impurity conduction vanishes. The dashed and full lines represent Sb- and As-doped germanium, respectively.

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[17] T. I. Pancove, in *Progress in Semiconductors*, edited by A. F. Gibson and R. E. Burgess (John Wiley & Sons, Inc., New York, to be published), Vol. 9.

[18] At 300°K the concentration dependence of the Hall mobility is $N^{-0.4}$ for As-doped germanium. Hence the discrepancy between electrical and optical measurements is less at the higher temperatures.

III. DISCUSSION

Previous analyses[5,6,8] of the transport properties of degenerate germanium were based on the simple degenerate model with the following assumptions.

1. The mobility components, $\mu_{\parallel}$ and $\mu_{\perp}$, parallel and perpendicular, respectively, to the valley axis, depend explicitly on concentration N and Fermi energy E_F, e.g.,

$$\mu \propto N^r E_F{}^s, \tag{1}$$

where the values of r and s may depend on the number of lower valleys and hence on stress because of screening effects and a changing contribution of intervalley and electron-electron scattering.

2. The bands are parabolic so that the Fermi energy E_F increases as

$$E_F \propto N^{2/3}. \tag{2}$$

With this the total N dependence of μ becomes

$$\mu \propto N^{r+2s/3}. \tag{3}$$

3. The total carrier concentration is independent of stress in the degenerate concentration region.

Assumption (1) yields for the low stress piezoresistance coefficient Π_{44}

$$\Pi_{44} = -\frac{1}{3}\frac{K-1}{2K+1}(\tfrac{3}{2}+s)\frac{E_2 S_{44}}{E_F}, \tag{4}$$

where the mobility anisotropy $K=\mu_{\perp}/\mu_{\parallel}$ and the value of s refer to the 4 valley case. E_2 is the shear deformation potential[1] and S_{44} the elastic compliance constant.

If the values of K, r, and s were independent of the distribution of the electrons over the valleys and hence of stress, then it would be a simple matter to determine these values from the low-stress and high-stress piezoresistance and from the N dependence of μ. This, however, is not the case as one can see immediately (Fig. 7) from the fact that $\mu(1)$, $\mu(2)$, and $\mu(4)$ depend differently on N. In this case further assumptions are required in order to analyze the data with the simple degenerate model.

Koenig and Katz[5] assume for the interpretation of their results that $r=-1.0$. With this one can obtain the value $s(4)$ for the 4 valley case from the N-dependence of $\mu(4)$ using Eq. (3), and subsequently one can obtain the value $K(4)$ from the low stress piezoresistance using Eq. (4). Katz also determined $K(4)$ from the stress dependence of the Hall coefficient at 77°K. He found that both methods yield a value of $K(4)$ between 2.1 and about 3.4 for an As concentration of $N=5\times10^{18}$ cm^{-3}.

The assumption $r=-1.0$ appears to be the weakest link in the chain of this analysis. For germanium in this concentration range Csavinszky's theory[7,6] of scattering at independent donor ions predicts for the 4 valley case $r(4)=-0.71$ because of the concentration-dependent screening effects. A failure of the individual scattering hypothesis is expected to result in even smaller magnitudes of r.

If we analyze our values of Π_{44} with Koenig's method but assume $r(4)=-0.72$ according to the results on Sb-doped Ge,[6] and if we use our value $r(4)+2s(4)/3=-0.22$, then we obtain for As-doped germanium $s(4)=0.75$ and $K(4)=4.0\pm0.4$.

The mobility anisotropy $K(4)$ has also been determined from magnetoresistance measurements,[19] particularly from the saturation of the longitudinal magnetoresistance.[20] Different authors[19,20] quote for As-doped germanium values increasing from $K=7$ to $K=9$ between $N=10^{18}$ and 10^{19} cm^{-3} and from $K=3.5$ to $K\approx6$ between $N=10^{18}$ and 6.2×10^{18} cm^{-3}. These data were taken near 77°K but at these high concentrations K should be nearly temperature-independent. Tsidilkovski *et al.*[20] quote an error of 10% for their magnetoresistance values. This gives rise to a 15–20% error in their determination of K. Furthermore, the value $K\approx6$ at $N=6.2\times10^{18}$ cm^{-3} was obtained from the low field magnetoresistance and is therefore even less certain. Our value $K(4)=4.0\pm0.4$ lies within these limits of uncertainty. The rather large K values quoted by Fistul *et al.*[20] were obtained from the low field magnetoresistance. They are again subject to the validity of the assumptions made in interpreting the low field results.

The values for $K(1)$ and $K(2)$ can in principle be obtained from the high stress longitudinal and transverse piezoresistance ratios without further assumptions. Because of the experimental difficulties with the transverse measurements, however, we have to assume in our case that at very high stress values the (111) conduction band valleys are sufficiently pushed apart so that effects of the stress inhomogeneities are unimportant. We then obtain for the mobility anisotropy in As-doped germanium $K(1)=6\pm0.5$ and $K(2)=5\pm0.6$ for the one and two valley cases, respectively. These values are considerably higher than $K(4)$ of Koenig and Katz[5] but much lower than a value obtained earlier[8] from an analysis of these data based on the simple degenerate model with intervalley scattering and the assumption that K is independent of the number of valleys.

Because the resistivity remains a second-rank tensor at all values of stress, one can obtain the principal axes of the mobility tensor $\mu_{\parallel}$ and $\mu_{\perp}$ for the one and two valley cases from the high stress longitudinal and transverse measurements without further assumptions. These values are shown in Fig. 8 as a function of donor concentration. The length of the vertical bars at the ends of the curves represent the error which stems pre-

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[19] D. G. Andrianov and V. I. Fistul, Fiz. Tverd. Tela **6**, 470 (1964) [English transl.: Soviet Phys.—Solid State **6**, 371 (1964)].

[20] I. M. Tsidilkovski, V. I. Sokolov, and G. I. Kharus, in *Proceedings of the International Conference on Semiconductor Physics, Paris, 1964* (Academic Press Inc., New York, 1964), p. 387; and V. I. Fistul, E. M. Omelyanovsky, D. G. Andrianov, and I. V. Dahovsky, *ibid.*, p. 371.